# 11.2 Conservation of Energy

Consider a ball near the surface of Earth. The sum of gravitational potential energy and kinetic energy in that system is constant. As the height of the ball changes, energy is converted from kinetic energy to potential energy, but the total amount of energy stays the same.

**► Objectives**

- **Solve** problems using the law of conservation of energy.
- **Analyze** collisions to find the change in kinetic energy.

**► Vocabulary**

law of conservation of energy
mechanical energy
thermal energy
elastic collision
inelastic collision

## Conservation of Energy

In our everyday world, it may not seem as if energy is conserved. A hockey puck eventually loses its kinetic energy and stops moving, even on smooth ice. A pendulum stops swinging after some time. The money model can again be used to illustrate what is happening in these cases.

Suppose you have a total of \$50 in cash. One day, you count your money and discover that you are \$3 short. Would you assume that the money just disappeared? You probably would try to remember whether you spent it, and you might even search for it. In other words, rather than giving up on the conservation of money, you would try to think of different places where it might have gone.

**Law of conservation of energy** Scientists do the same thing as you would if you could not account for a sum of money. Whenever they observe energy leaving a system, they look for new forms into which the energy could have been transferred. This is because the total amount of energy in a system remains constant as long as the system is closed and isolated from external forces. The **law of conservation of energy** states that in a closed, isolated system, energy can neither be created nor destroyed; rather, energy is conserved. Under these conditions, energy changes from one form to another while the total energy of the system remains constant.

**Conservation of mechanical energy** The sum of the kinetic energy and gravitational potential energy of a system is called **mechanical energy.** In any given system, if no other forms of energy are present, mechanical energy is represented by the following equation.

**Mechanical Energy of a System** $E = KE + PE$

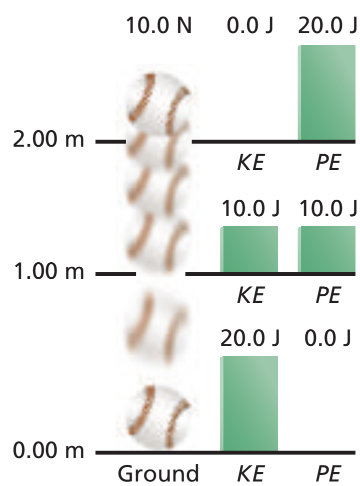
The mechanical energy of a system is equal to the sum of the kinetic energy and potential energy if no other forms of energy are present.

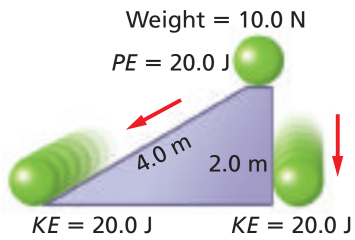
Imagine a system consisting of a 10.0-N ball and Earth, as shown in **Figure 11-9.** Suppose the ball is released from 2.00 m above the ground, which you set to be the reference level. Because the ball is not yet moving, it has no kinetic energy. Its potential energy is represented by the following equation:

$$PE = mgh = (10.0\ \text{N})(2.00\ \text{m}) = 20.0\ \text{J}$$

The ball's total mechanical energy, therefore, is 20.0 J. As the ball falls, it loses potential energy and gains kinetic energy. When the ball is 1.00 m above Earth's surface: $PE = mgh = (10.0\ \text{N})(1.00\ \text{m}) = 10.0\ \text{J}$.

■ **Figure 11-9** A decrease in potential energy is equal to the increase in kinetic energy.






■ **Figure 11-10** The path that an object follows in reaching the ground does not affect the final kinetic energy of the object.

What is the ball's kinetic energy when it is at a height of 1.00 m? The system consisting of the ball and Earth is closed and isolated because no external forces are acting upon it. Hence, the total energy of the system, $E$, remains constant at 20.0 J.

$$E = KE + PE, \text{ so } KE = E - PE$$

$$KE = 20.0 \text{ J} - 10.0 \text{ J} = 10.0 \text{ J}$$

When the ball reaches ground level, its potential energy is zero, and its kinetic energy is 20.0 J. The equation that describes conservation of mechanical energy can be written as follows.

**Conservation of Mechanical Energy**

$$KE_{before} + PE_{before} = KE_{after} + PE_{after}$$

When mechanical energy is conserved, the sum of the kinetic energy and potential energy present in the system before the event is equal to the sum of the kinetic energy and potential energy in the system after the event.

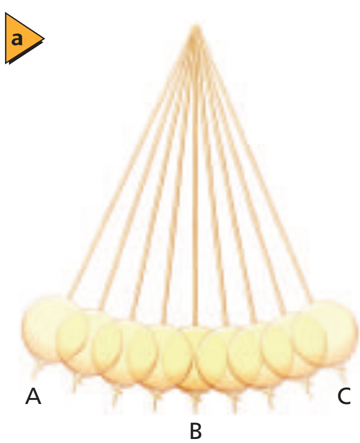
What happens if the ball does not fall down, but rolls down a ramp, as shown in **Figure 11-10?** If there is no friction, there are no external forces acting on the system. Thus, the system remains closed and isolated. The ball still moves down a vertical distance of 2.00 m, so its loss of potential energy is 20.0 J. Therefore, it gains 20.0 J of kinetic energy. As long as there is no friction, the path that the ball takes does not matter.

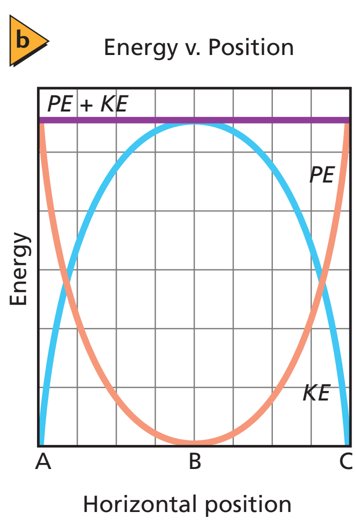
**Roller coasters** In the case of a roller coaster that is nearly at rest at the top of the first hill, the total mechanical energy in the system is the coaster's gravitational potential energy at that point. Suppose some other hill along the track were higher than the first one. The roller coaster would not be able to climb the higher hill because the energy required to do so would be greater than the total mechanical energy of the system.

**Skiing** Suppose you ski down a steep slope. When you begin from rest at the top of the slope, your total mechanical energy is simply your gravitational potential energy. Once you start skiing downhill, your gravitational potential energy is converted to kinetic energy. As you ski down the slope, your speed increases as more of your potential energy is converted to kinetic energy. In ski jumping, the height of the ramp determines the amount of energy that the jumper has to convert into kinetic energy at the beginning of his or her flight.

**Pendulums** The simple oscillation of a pendulum also demonstrates conservation of energy. The system is the pendulum bob and Earth. Usually, the reference level is chosen to be the height of the bob at the lowest point, when it is at rest. If an external force pulls the bob to one side, the force does work that gives the system mechanical energy. At the instant the bob is released, all the energy is in the form of potential energy, but as the bob swings downward, the energy is converted to kinetic energy. **Figure 11-11** shows a graph of the changing potential and kinetic energies of a pendulum. When the bob is at the lowest point, its gravitational potential energy is zero, and its kinetic energy is equal to the total mechanical

■ **Figure 11-11** For the simple harmonic motion of a pendulum bob **(a),** the mechanical energy—the sum of the potential and kinetic energies—is a constant **(b).**



energy in the system. Note that the total mechanical energy of the system is constant if we assume that there is no friction. You will learn more about pendulums in Chapter 14.

**Loss of mechanical energy** The oscillations of a pendulum eventually come to a stop, a bouncing ball comes to rest, and the heights of roller-coaster hills get lower and lower. Where does the mechanical energy in such systems go? Any object moving through the air experiences the forces of air resistance. In a roller coaster, there are frictional forces between the wheels and the tracks.

When a ball bounces off of a surface, all of the elastic potential energy that is stored in the deformed ball is not converted back into kinetic energy after the bounce. Some of the energy is converted into thermal energy and sound energy. As in the cases of the pendulum and the roller coaster, some of the original mechanical energy in the system is converted into another form of energy within members of the system or transmitted to energy outside the system, as in air resistance. Usually, this new energy causes the temperature of objects to rise slightly. You will learn more about this form of energy, called **thermal energy,** in Chapter 12. The following strategies will be helpful to you when solving problems related to conservation of energy.

## PROBLEM-SOLVING Strategies

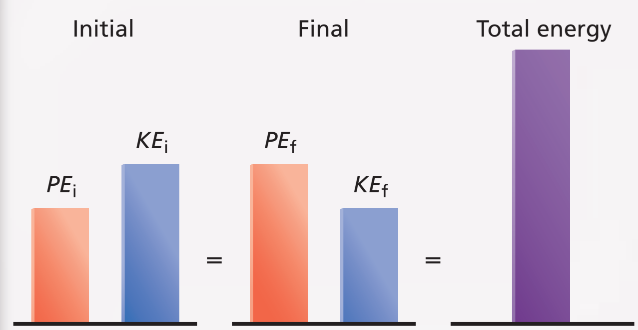
### Conservation of Energy

*When solving problems related to the conservation of energy, use the following strategies.*

1. Carefully identify the system. Make sure it is closed. In a closed system, no objects enter or leave the system.
2. Identify the forms of energy in the system.
3. Identify the initial and final states of the system.
4. Is the system isolated?
   a. If there are no external forces acting on the system, then the system is isolated and the total energy of the system is constant.
   $$E_{before} = E_{after}$$
   b. If there are external forces, then the following is true.
   $$E_{before} + W = E_{after}$$
5. If mechanical energy is conserved, decide on the reference level for potential energy. Draw bar graphs showing initial and final energy like the bar graphs shown to the right.

Connecting Math to Physics

### Energy Bar Graphs

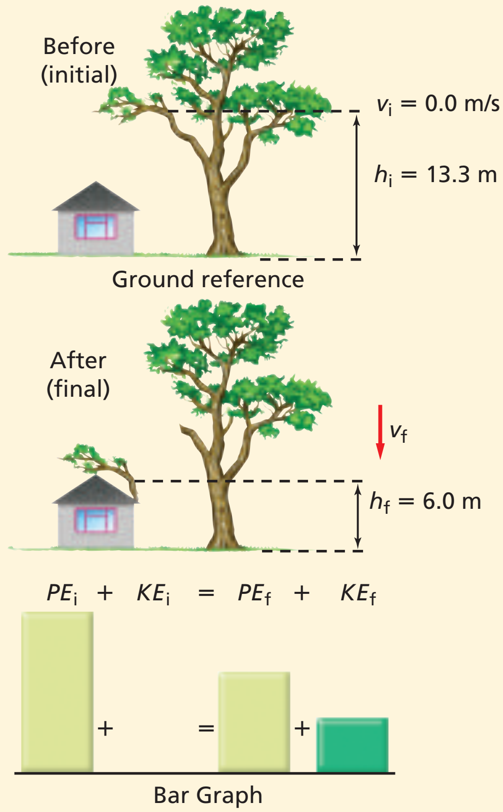


# EXAMPLE Problem 2

**Conservation of Mechanical Energy** During a hurricane, a large tree limb, with a mass of 22.0 kg and a height of 13.3 m above the ground, falls on a roof that is 6.0 m above the ground.

**a.** Ignoring air resistance, find the kinetic energy of the limb when it reaches the roof.

**b.** What is the speed of the limb when it reaches the roof?




## 1 Analyze and Sketch the Problem

- Sketch the initial and final conditions.
- Choose a reference level.
- Draw a bar graph.

**Known:**

$m = 22.0$ kg　　$g = 9.80$ m/s$^2$
$h_{limb} = 13.3$ m　　$v_i = 0.0$ m/s
$h_{roof} = 6.0$ m　　$KE_i = 0.0$ J

**Unknown:**

$PE_i = ?$　　$KE_f = ?$
$PE_f = ?$　　$v_f = ?$

## 2 Solve for the Unknown

**a.** Set the reference level as the height of the roof.

Solve for the initial height of the limb relative to the roof.

$$h = h_{limb} - h_{roof}$$
$$= 13.3 \text{ m} - 6.0 \text{ m}$$　　**Substitute $h_{limb}$ = 13.3 m, $h_{roof}$ = 6.0 m**
$$= 7.3 \text{ m}$$

Solve for the initial potential energy of the limb.

$$PE_i = mgh$$
$$= (22.0 \text{ kg})(9.80 \text{ m/s}^2)(7.3 \text{ m})$$　　**Substitute $m$ = 22.0 kg, $g$ = 9.80 m/s$^2$, $h$ = 7.3 m**
$$= 1.6\times10^3 \text{ J}$$

Identify the initial kinetic energy of the limb.

$$KE_i = 0.0 \text{ J}$$　　**The tree limb is initially at rest.**

The kinetic energy of the limb when it reaches the roof is equal to its initial potential energy because energy is conserved.

$$KE_f = PE_i$$　　**$PE_f$ = 0.0 J because $h$ = 0.0 m at the reference level.**
$$= 1.6\times10^3 \text{ J}$$

**b.** Solve for the speed of the limb.

$$KE_f = \frac{1}{2}mv_f^2$$
$$v_f = \sqrt{\frac{2KE_f}{m}}$$
$$= \sqrt{\frac{2(1.6\times10^3 \text{ J})}{22.0 \text{ kg}}}$$　　**Substitute $KE_f$ = 1.6×10$^3$ J, $m$ = 22.0 kg**
$$= 12 \text{ m/s}$$

**Math Handbook**
Square and Cube Roots
pages 839–840

## 3 Evaluate the Answer

- **Are the units correct?** Velocity is measured in m/s and energy is measured in kg·m$^2$/s$^2$ = J.
- **Do the signs make sense?** *KE* and the magnitude of velocity are always positive.